NUMERICAL COMPUTATION OF THE CAPACITANCE FOR THE SPHERICAL CAPACITOR

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Abstract: Numerical computation methods are frequently introduced to the undergraduate Electrical Engineering student in the course in electromagnetic theory. In this note, we demonstrate that a spherical capacitor can firstly be solved analytically and then compared with an easily model obtained by numerical solution [2]. In this study, a three-dimensional model represents a capacitance for the spherical capacitor.

Keywords: numerical calculation, spherical capacitor, electrostatic field

1. THE ANALYTICAL METHOD

The capacitance for spherical or cylindrical conductors can be obtained by evaluating the voltage difference between the conductors for a given charge on each. By applying Gauss' Law to a charged conducting sphere, the electric field outside it is found to be [6],[10]:

\[ E = \frac{Q}{4\pi \varepsilon_0 r^2} \]  

The voltage between the spheres can be found by integrating the electric field along a radial line:

\[ \Delta V = \int_a^b \frac{Q}{4\pi \varepsilon_0 r^2} \frac{1}{r} \, dr = \frac{Q}{4\pi \varepsilon_0} \left[ \frac{1}{a} - \frac{1}{b} \right] \]  

The outer shell with radius \( b \) is carrying a charge \(-Q\), and the inner sphere of radius \( a \) is carrying a charge \(+Q\) (Figure 1). From the definition of capacitance, the capacitance is:

\[ C = \frac{Q}{\Delta V} = \frac{4\pi \varepsilon_0}{\frac{1}{a} - \frac{1}{b}} \]  

A solid conducting sphere of radius \( R \) carries a charge \( Q \). Computing the electric field energy density at a point a distance \( r \) from the center of the sphere for \( r<R \) and \( r>R \).

The electric field energy density at a point a distance \( r \) from the center of a conducting sphere of radius \( R \) that carries total charge \( Q \) is [6],[11]:

\[ w = \frac{1}{2} \varepsilon_0 E^2 = 0 \]  

since for \( r<R \) the electric field is zero (inside the conducting sphere).

The permittivity is usually expressed as the product of a relative permittivity \( \varepsilon_r \), and the permittivity of free space \( \varepsilon_0 \):

\[ \varepsilon = \varepsilon_r \cdot \varepsilon_0 \]  

where \( \varepsilon_0 = 8.854 \cdot 10^{-12} \frac{F}{m} \).

The relative permittivity is unity for a vacuum and typically in the range of 2 to 6 for most dielectrics, as we shall discuss in more detail later.

2. THE NUMERICAL CALCULATION METHOD

The CST EM STUDIO is used for computing the capacitance matrix for the spherical capacitor,
electrostatic field distribution, electric induction distribution, electric field energy and charge [4].

CST EM STUDIO is a fully featured software package for electromagnetic analysis and design of electrostatic, magnetostatic and low-frequency devices based on the Finite Integration Technique (FIT). This numerical method provides an universal spatial discretization scheme applicable to various electromagnetic problems ranging from static field computations to high frequency applications in time or frequency domain [4].

The spherical capacitor example has been designed to demonstrate the parameter sweep feature in combination with the capacitance calculation.

Use Gauss’ Law for two spherical shells, one of radius $r$ and the other of radius $r+dr$. Find the charge contained in the infinitesimal spherical shell of thickness $dr$.

The structure consists of a small sphere of perfectly conducting material (PEC), embedded in a larger vacuum sphere. The background material is set to PEC. Three tangential symmetry conditions are used to reduce the calculation domain and therefore the calculation time [4]. Furthermore a parameter $dr$ was defined representing the difference of the radii of the inner and outer sphere (Figure 2).

![Figure 2. The spherical capacitor.](image)

To perform electrostatic calculations a potential value of 10V is defined at the inner sphere. The perfectly conducting background material is set to 0V by default (Figure 3).

The computing domain is covered and terminated by six faces. The boundary conditions must be assigned to each of these faces. For this problem, we can define a symmetry condition for the XY, XZ and YZ planes. This will reduce the computation time by factor of 4, so symmetry conditions should always be defined, if it is possible (Figure 4).

![Figure 3. The potential values.](image)

![Figure 4. The boundary conditions.](image)

Generally, there are three ways to define a hexahedral mesh: manually, automatically and adaptively. In this problem, the mesh is define automatically. This is probably the most effective way of working with CST STUDIO SUITE mesh generator determines the important features of your structure and automatically creates a mesh, which represents your structure and the fields equally well. This means that the frequency range and dielectrics, metallic edges, etc. are considered by the expert system, but certain mesh properties for individual shapes can also be set manually. At the first level, this mesh generator is governed by only a few settings, but there are many other possibilities to influence the meshing [4].

In CST EM STUDIO several different material properties are considered to allow realistic modeling of practical simulation problems. The two basic materials available are PEC (Perfect Electrically Conducting material) and Vacuum, other more complex materials can be defined in the Material Parameters dialog. Each material is distinguished by its unique name and can be visualized in a selectable color and transparency [4].

By dielectric material:

- **Vacuum**: Two metal, usually copper, electrodes are separated by a vacuum. The insulating envelope is usually glass or ceramic. Typically of low capacitance - 10 - 1000 pF and high voltage, up to tens of kilovolts, they are most often used in radio transmitters and other high voltage power devices. Both fixed and variable types are available. Vacuum variable capacitors can have a minimum to maximum capacitance ratio of up to 100, allowing any tuned circuit to cover a full decade of frequency. Vacuum is the most perfect of dielectrics with a zero loss tangent. This allows very high powers to be transmitted without significant loss and consequent heating [1],[3].

- **Air**: Air dielectric capacitors consist of metal plates separated by an air gap. The metal plates, of which there may be many interleaved, are most often made of aluminium or silver-plated brass. Nearly all air dielectric capacitors are variable and are used in radio tuning circuits.
• **Plastic film**: Made from high quality polymer film (usually polycarbonate, polystyrene, polypropylene, polyester (Mylar), and for high quality capacitors polysulfone), and metal foil or a layer of metal deposited on surface of the plastic film in the metalized film type. They have good quality and stability, and are suitable for timer circuits. Their inductance limits use at high frequencies [1].

• **Mica**: Similar to glass. Often high voltage. Suitable for high frequencies. Expensive. Excellent tolerance & stability.

• **Paper**: Used for relatively high voltages. Known for long term failures.

• **Glass**: Used for high voltages. Expensive. Stable temperature coefficient in a wide range of temperatures.

• **Ceramic**: Chips of alternating layers of metal and ceramic, or disks of ceramic with metal on both sides of the disk. Characteristics vary widely depending on the type of ceramic dielectric. The dielectrics are broadly categorized as Class 1 or Class 2. Class 2 ceramic capacitors have strong variation of capacitance with temperature, high dissipation factor, high frequency coefficient of dissipation, and their capacitance depends on applied voltage and changes with aging. However they find massive use in common low-precision coupling and filtering applications. Suitable for high frequencies [8].

• **Aluminum electrolytic**: One electrode made of aluminum foil, etched aluminum to acquire much larger surface area. The dielectric is oxide grown on the etched aluminum plate, and the second electrode is a liquid electrolyte. They can achieve high capacitance but suffer from poor tolerances, high instability, gradual loss of capacitance especially when subjected to heat, and high leakage current. The conductivity of the electrolyte drops at low temperatures, increasing equivalent series resistance. Bad frequency characteristics make them unsuited for high-frequency applications. Special types with low equivalent series resistance are available [9].

• **Tantalum electrolytic**: Similar to the aluminum electrolytic capacitor but with better frequency and temperature characteristics. High dielectric absorption and high leakage [3]. Although they share many of the disadvantages of aluminum electrolytics, they perform better on most attributes; for example, they have much better performance at low temperatures [7].

In the first application, we considered the spherical capacitor with different dielectric materials. The dielectric materials are: vacuum, polystyrene, mica and waxed paper. The relative permittivity for vacuum is 1, for polystyrene is 2.55, for mica is 6 and for waxed paper is 3.5. The inner radius is 1 mm and the outer radius is 1.5 mm.

Furthermore, a parameter "dr" was defined representing the difference of the radii of the inner and outer sphere.

The parameter "dr" is considered for variation between 0.5 and 5 using the parameter sweep feature. Running the parameter sweep the capacitance matrix of the spherical capacitor is monitored [4]. The Figure 5 shows the influence of the parameter "dr" on the capacitance of the structure in case waxed paper dielectric.

The Figure 6 shows the distribution of electric field strength and the Figure 7 shows the distribution of flux density for spherical capacitor with vacuum dielectric.

Hence we have obtained the following results (Table 1):

<table>
<thead>
<tr>
<th>Electrical measures</th>
<th>Dielectric material</th>
<th>Vacuum</th>
<th>Polystyrene</th>
<th>Mica</th>
<th>Wax Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance [F]</td>
<td></td>
<td>3.352</td>
<td>10&lt;sup&gt;-11&lt;/sup&gt;</td>
<td>8.509</td>
<td>10&lt;sup&gt;-12&lt;/sup&gt;</td>
</tr>
<tr>
<td>Electric field strength [V/m]</td>
<td>34513,3</td>
<td>33579,4</td>
<td>32844,8</td>
<td>33309,7</td>
<td></td>
</tr>
<tr>
<td>Flux density [C/m&lt;sup&gt;2&lt;/sup&gt;]</td>
<td>3.0558</td>
<td>10&lt;sup&gt;-7&lt;/sup&gt;</td>
<td>7.5816</td>
<td>10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>1.7448</td>
</tr>
<tr>
<td>Electric field energy [J]</td>
<td>1.675</td>
<td>10&lt;sup&gt;-12&lt;/sup&gt;</td>
<td>4.253</td>
<td>10&lt;sup&gt;-11&lt;/sup&gt;</td>
<td>1.004</td>
</tr>
<tr>
<td>Charge [C]</td>
<td>3.352</td>
<td>10&lt;sup&gt;-12&lt;/sup&gt;</td>
<td>8.509</td>
<td>10&lt;sup&gt;-12&lt;/sup&gt;</td>
<td>2.009</td>
</tr>
</tbody>
</table>

**Figure 5. The influence of the parameter "dr" on the capacitance of the structure.**

**Figure 6. The distribution of electric field strength.**
In the second application, we considered the spherical capacitor with dielectric material vacuum and different inner sphere radius. The parameter "dr" is considered fix 0.5 (Table 2).

<table>
<thead>
<tr>
<th>Electrical measures</th>
<th>The inner sphere radius</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 mm</td>
</tr>
<tr>
<td>Capacitance [F]</td>
<td>0.335 10^{-12}</td>
</tr>
<tr>
<td>Electric field strength [V/m]</td>
<td>34513.3</td>
</tr>
<tr>
<td>Flux density [C/m²]</td>
<td>3.058 10^{-7}</td>
</tr>
<tr>
<td>Electric field energy [J]</td>
<td>1.675 10^{-11}</td>
</tr>
<tr>
<td>Charge [C]</td>
<td>3.352 10^{-12}</td>
</tr>
</tbody>
</table>

In the third application, we considered the spherical capacitor with vacuum dielectric, inner radius sphere of 4 mm, outer radius sphere of 4.5 mm and potential 20 V. Hence we have obtained the following results:
- Capacitance Matrix: 4.059057 10^{-12} [F]
- Electric field energy: 8.118196 10^{-10} [J]
- Charge: 8.118113 10^{-11} [C]
- Electric field strength: 50683.6 [V/m]
- Flux density: 4.48762 10^{-7} [C/m²]

The Figure 10 shows the distribution of electric field strength and the Figure 11 shows the distribution of flux density for spherical capacitor with vacuum dielectric and potential 20 V.

3. CONCLUSIONS

Capacitors are used in almost every activity of electrical engineering, yet information on capacitor characteristics is printed through a variety of textbooks, databooks, and manufacturers literature.

These two steps are typically for each analysis procedure, which can be:
- analytical;
- numerical.

This numerical application aided for quick computation electrical measures for spherical capacitor.

REFERENCES

[1] Zorpette, G. Super Charged: A Tiny South Korean Company is Out to Make Capacitors Powerful enough to Propel the Next Generation


[8] Kemet Ceramic leaded Capacitors F-3101F 06/05. See also: http://www.kemet.com

